Factorial

```
J=1;
Fact=1;
while (J<N) {
  J=J+1;
  Fact:=Fact*J;
}
```

The condition which holds before the loop, after each iteration of the loop and at the end of the loop is called *loop invariant*.

Loop invariants -properties

- The loop invariant when combined with the termination condition gives the desired result at the end of the loop
- The loop invariant must be true at the entry point of the loop and at the end of the loop body.

Loop invariants - discovery

1. Understand the loop strategy used in the implementation
2. Express the desired result at the exit of the loop in terms of loop variables and loop exit condition.

Greatest Common Divisor gcd

Fix our universe to natural numbers. x is a *divisor* of y if there exists z such that $y = x * z$.

d is a *common divisor* of m and n if it is a divisor of both m and n.

The *greatest common divisor* of m and n, written $\gcd(m, n)$ is a common divisor of m and n such that any other common divisor d of m and n is also a divisor of $\gcd(m, n)$.

Exercise: Prove that if M and N are naturals then, upon the execution of the above program, $y = \gcd(M, N)$.

```
x=M;y=N;
while (x>0) {
  if (x>=y) x=x-y;
  else {t=x; x=y; y=t;}
}
```

Bubble Sort

Exercise: Prove that given an array $S[1 \ldots N]$, $N \geq 1$, the following program puts in array s the elements of S, in order.

```
s=S; sorted=0;
while (sorted!=1) {
  sorted=1;
  i=1;
  while (i<N) {
    if (s[i]>s[i+1]) {
      sorted=0;
      t=s[i]; s[i]=s[i+1]; s[i+1]=t;
    }
    i=i+1;
  }
}
```